Homework 1

I. Convergence Analysis

Consider the wave equation:

$$u_t + au_x = 0.$$

Establish whether or not the following methods for solving the equation converge. If so, what are the conditions for convergence. **Hint**: Use the Lax-Richtmyer equivalence theorem. Chapters 1 and 2 of the text by Strikwerda will be helpful.

A. Explicit Central Differencing

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} + a\frac{v_{j+1}^n - v_{j-1}^n}{2\Delta x} = 0.$$

B. Implicit Central Differencing

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} + a\frac{v_{j+1}^{n+1} - v_{j-1}^{n+1}}{2\Delta x} = 0.$$

C. Upwinding

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} + aD^* v_j^n = 0$$

If $a > 0$, $D^* = D^-$. If $a < 0$, $D^* = D^+$.

D. Downwinding

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} + aD^* v_j^n = 0$$

If $a > 0$, $D^* = D^+$. If $a < 0$, $D^* = D^-$.