Homework 2

I. LAX-RICHTMYER THEOREM

Recall the standard wave equation,

$$u_t + au_x = 0,$$

where $a = constant$. Prove that stability and consistency are sufficient for convergence for approximations to the wave equation. See Chapter 8 of *Finite Volume Methods for Hyperbolic Problems*, by Randall J. LeVeque (there is a link to an electronic copy of the book on the class website).

II. TVD

Define the 'total variation' of v as

$$TV(v) = \sum_{j=1}^{N} |v_{j+1} - v_j|.$$

Prove that second order Runge-Kutta is total variation diminishing in the sense that $TV(v^{n+1}) <= TV(v^n)$. You may assume that the forward Euler step is TVD.