CS 237C(CME 306)
Midterm I

May 2, 2005

Name	
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Question	Points	Score
1	15	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	25	
11	20	
Total	100	

1. Indicate whether each statement is true or false.

   T / F Unlike shocks, contact discontinuities travel at the local characteristic speed.

   T / F The weak solution of a conservation law is unique.

   T / F The vanishing viscosity solution of a conservation law converges to the entropy satisfying weak solution of the conservation law.

   T / F Consider a weak solution for Burgers equation. In the interior of a rarefaction wave, the solution also satisfies the strong form.

   T / F A high resolution method refers to a method where, almost everywhere in the domain, the dominant error term is high order, even if the method has a low order of accuracy.

   T / F A linear, second order accurate numerical method is stable if and only if it is convergent.

   T / F The Lax-Wendroff theorem tells us under which conditions a numerical approximation can be expected to converge.

2. Let $\mathcal{N}$ denote a consistent, linear, one-step numerical method. Which of the following statements are true?

   I. If $\|\mathcal{N}\| > 1$ then the method is unstable.

   II. If the method yields a numerical solution whose norm grows exponentially in time then the method is unstable.

   III. If the method does not satisfy the CFL condition then the method is unstable.

   (a) I only

   (b) II only

   (c) III only

   (d) II and III only

   (e) I, II, and III

3. The ENO-RF method is preferable to the ENO-Roe method for computing the numerical flux function because ENO-Roe

   (a) results in too much dissipation in the numerical solution.
   (b) may create spurious oscillations in the vicinity of shocks and other nonlinear phenomena.
   (c) may compute a nonphysical weak solution.
   (d) has a lower order of accuracy.
   (e) can be shown to diverge in some cases where ENO-RF converges.

4. Which statement is true?

   (a) Second order TVD Runge-Kutta is an improvement upon second order Runge-Kutta because the latter is not always TVD.
   (b) The exact solution of the scalar, constant coefficient advection equation has total variation which decreases in time. Therefore, TVD schemes which mimic this behavior are desirable.
   (c) If we have a spatial discretization such that a forward Euler step is TVB, then third order TVD Runge-Kutta in conjunction with that spatial discretization is TVB.
   (d) If we have a spatial discretization such that a forward Euler step is TVB, then third order TVD Runge-Kutta in conjunction with that spatial discretization is TVD.
   (e) None of the above.

5. Consider the first order upwind method for the scalar, constant coefficient advection equation $u_t + au_x = 0$. Which statement is true?

   (a) Modified equation analysis for the method indicates that the numerical solution will exhibit dispersive behavior.
   (b) In exact arithmetic, if we take $\frac{\triangle x}{\triangle t} = |a|$, the method can yield the exact solution.
   (c) The method is unconditionally stable.
   (d) The method is preferable to explicit central differencing, although both methods have the same order of accuracy.
   (e) None of the above.

6. Consider the scalar conservation law $u_t + \left(\frac{u^2}{2}\right)_x = 0$. Which of the following statements are necessarily true?

   I. The characteristic curves are straight lines.

   II. Shocks travel in straight lines.

   III. Shocks cannot form if the initial data is smooth.

   (a) I only
   (b) II only
   (c) III only
   (d) I and II only
   (e) I, II, and III

7. Which of the following schemes for Burgers equation can be written in discrete conservation form?

   I. $\frac{u_j^{n+1}-u_j^n}{\triangle t} + u_j^n \left(\frac{u_{j+1}^n - u_j^n}{\triangle x}\right) = 0$

   II. $\frac{u_j^{n+1}-u_j^n}{\triangle t} + u_j^n \left(\frac{u_{j+1}^n - u_{j-1}^n}{2\triangle x}\right) = 0$

   III. $\frac{u_j^{n+1}-u_j^n}{\triangle t} + \frac{1}{\triangle x}\left(\frac{1}{2}\left(u_j^n\right)^2 - \frac{1}{2}\left(u_{j-1}^n\right)^2\right) = 0$

   (a) None
   (b) III only
   (c) II and III only
   (d) I and II only
   (e) I and III only

8. ENO schemes construct an interpolating polynomial in order to

   (a) use higher order interpolation near steep gradients and discontinuities.
   (b) create a numerical stencil that avoids crossing shocks.
   (c) allow for higher order in time approximations to u_t.
   (d) remain conservative.
   (e) none of the above

9. Consider a consistent, stable, linear, one-step numerical method. Which of the following statements are true?

    I. Local truncation errors accumulate at each time step.

    II. Local truncation errors accumulate and may be amplified at each time step.

    III. In order to get convergence to the exact solution, we must choose the initial data for the scheme to be equal pointwise to the initial data for the exact solution.

    (a) I only

    (b) III only

    (c) I and II only

    (d) I and III only

    (e) I, II and III

10. Consider the scalar, constant coefficient advection equation

$$u_t + au_x = 0$$

and the following numerical method for computing an approximate solution to it.

$$u_j^{n+1} = \frac{1}{2}\left(u_{j-1}^n + u_{j+1}^n\right) - \frac{\triangle t}{2\triangle x} a \left(u_{j+1}^n - u_{j-1}^n\right)$$

Analyze the stability and consistency of the method. What is the order of accuracy of the method? What can you say about convergence of the method? You may assume that $\frac{\triangle t}{\triangle x} = constant$.

11. Consider the scalar, constant coefficient advection equation

$$u_t + au_x = 0$$

Show that for this equation, the ENO-Roe and ENO-LLF schemes will produce identical numerical approximations.