Homework 8

I.

Consider the heat equation in $1D$,

$$u_t = bu_{xx}.$$

1) Under which conditions are each of the schemes below stable? Justify your answers.
2) For each scheme, state the order of accuracy in time and space (you don't need to prove this, just state it).

A. Forward Euler

$$\frac{u_j^{n+1} - u_j^n}{\triangle t} = b \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\triangle x^2}$$

B. Backward Euler

$$\frac{u_j^{n+1} - u_j^n}{\triangle t} = b \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{\triangle x^2}$$

C. Crank-Nicholson

$$\frac{u_j^{n+1} - u_j^n}{\triangle t} = \frac{b}{2} \left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\triangle x^2} + \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{\triangle x^2} \right)$$