CS 237C(CME 306)
Midterm II

May 30, 2006

Name	
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Question	Points	Score
1	5.5	
2	5.5	
3	5.5	
4	5.5	
5	5.5	
6	5.5	
7	5.5	
8	5.5	
9	20	
10	36	
Total	100	

1. Consider the projection method for incompressible flow. Which of the following statements are true?

   I. An elliptic equation is solved for the pressure, which is then used in computing the divergence free velocity field.

   II. The time step for the method is typically restricted by the update for $\vec{V}^{n+1}$ in the last step.

   III. The computation yields a Helmholtz-Hodge decomposition of $\vec{V}^{\star}$.

   (a) I only

   (b) II only

   (c) III only

   (d) I and II only

   (e) I and III only

2. In discretizing the equation $u_t = u_{xx}$,

   (a) Crank-Nicholson is a good method to use, since it is unconditionally stable and gives the steady-state solution as $\triangle t \to \infty$.

   (b) if we use a consistent method of the form

   $$u_j^{n+1} = \alpha u_{j-1}^n + \beta u_j^n + \gamma u_{j+1}^n$$

   where α, β, and γ are constants the CFL condition is violated if $\triangle t = \triangle x$.

   (c) we don't have to worry about a CFL condition since the equation is parabolic.

   (d) the backward Euler scheme is a popular choice because it is unconditionally stable and second order accurate.

   (e) None of the above.

3. For the Neumann problem for Poisson's equation,

   (a) a solution to the problem may not exist.

   (b) if the problem satisfies the compatibility condition, then a unique solution exists.

   (c) a solution exists, but it is unique only up to a constant.

   (d) we can use a second order accurate discretization to get a symmetric negative definite linear system.

   (e) None of the above.

4. Consider the projection method for viscous incompressible flow equations. Which of the following statements are true?

   I. Viscosity is included in the computation of the intermediate velocity field.

   II. Explicit discretization of the viscosity imposes an undesirable time step restriction of $\triangle t = O(\triangle x^2)$.

   III. The numerical viscosity is typically negligible compared to the physical viscosity.

   (a) I only
   (b) II only
   (c) I and III only
   (d) I and II only
   (e) None

5. The main advantage of the semi-Lagrangian scheme over an ENO or upwind scheme, is that the semi-Lagrangian scheme

   (a) is higher order.
   (b) suffers from less numerical viscosity.
   (c) is unconditionally stable.
   (d) more accurately approximates shock propagation speeds.
   (e) None of the above.

6. When solving a system of conservation laws using ENO-LLF,

   (a) we use left eigenvectors of $J(\vec{U}_j)$ to transform the system into the N characteristic fields for each grid point j.

   (b) $\vec{F}_{i+\frac{1}{2}} = R \begin{pmatrix} F^1_{i+\frac{1}{2}} \\ \vdots \\ F^N_{i+\frac{1}{2}} \end{pmatrix}$, where $F^p_{i+\frac{1}{2}}$ is the scalar numerical flux for the p-th characteristic field, R is the matrix of right eigenvectors of $J(\vec{U}_{i+\frac{1}{2}})$, and $(\vec{U}_i)_t + \frac{\vec{F}_{i+\frac{1}{2}} - \vec{F}_{i-\frac{1}{2}}}{\triangle x} = 0$.

   (c) we multiply the flux by a left eigenvector in order to determine the 0^{th} row of our divided difference table.

   (d) the method depends on the fact that the matrix $L_0 J R_0$ is exactly diagonalized in a neighborhood of x_0.

   (e) the upwind directions for the characteristic fields do not vary in space or time.

7. Consider a system of m equations

$$\vec{U}_t + \vec{F}(\vec{U})_x = 0, \tag{1}$$

that is hyperbolic at each point $(x,t) \in \mathbb{R} \times [0,\infty)$ and let

$$A(\vec{U}) = \frac{\partial \vec{F}}{\partial \vec{U}}.$$

Which of the following statements are true?

I. If A is constant, we can decouple the system (1) into m scalar, constant coefficient equations.

II. The CFL condition for the system (1) is based on the characteristic velocities of the corresponding linearized system.

III. $A(\vec{U})$ has m real eigenvalues and m linearly independent left eigenvectors $\forall \vec{U} \in \mathbb{R}^m$.

(a) I and II only

(b) I and III only

(c) III only

(d) I, II, and III

(e) None

8. In modeling a flow as incompressible,

(a) we must specify an equation of state for the pressure in order to get a closed system.

(b) we can get a more favorable time step restriction than for compressible flow.

(c) we must solve for mass, momentum, and energy simultaneously since the equations are coupled.

(d) we are assuming that the sound speed is slow relative to the fluid velocity.

(e) None of the above.

9. Consider the projection method for incompressible flow, where the steps are given below.

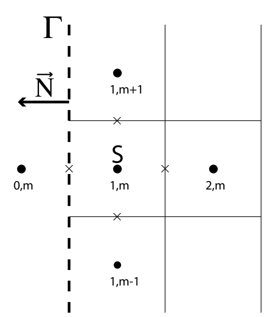
$$\frac{\vec{V}^\star - \vec{V}^n}{\triangle t} + \vec{V}^n \cdot \nabla \vec{V}^n = \vec{g} \tag{2}$$

$$\triangle \hat{p}^{n+1} = \nabla \cdot \vec{V}^\star \tag{3}$$

$$\vec{V}^{n+1} - \vec{V}^\star + \nabla \hat{p}^{n+1} = 0 \tag{4}$$

(a) Using equations (3) and (4), write down the Neumann problem for $\hat{p}^{n+1}$. Do <u>not</u> assume that $\vec{V}^\star = \vec{V}^{n+1}$ on the boundary.

(b) Consider the figure below, which depicts a MAC grid containing a point S which lies next to the boundary Γ, with outward unit normal $\vec{N} = (-1, 0)$.



Using standard second order accurate central differencing for all derivatives, discretize the equation for $\hat{p}^{n+1}$ at the point S, applying the boundary condition.

(c) Explain why your discretization at the point S suggests that we can set $\vec{V}^\star\Big|_\Gamma = \vec{V}^{n+1}\Big|_\Gamma$ without affecting the solution $\hat{p}^{n+1}$.

10. Find the exact solution of the system

$$\begin{cases} \begin{pmatrix} u \\ \phi \end{pmatrix}_t + \begin{pmatrix} a & 1 \\ c^2 & a \end{pmatrix} \begin{pmatrix} u \\ \phi \end{pmatrix}_x = 0 \\ u(x,0) = u_0(x) \\ \phi(x,0) = \phi_0(x) \end{cases}$$

where a and c are real constants and $c \neq 0$.